Implementation and Assessment of On-line Gateway Testing[1] at the University of Michigan

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1 Abstract

Gateway tests provide a means of assuring that students in reformed precalculus and calculus courses acquire the algebraic and computational skills needed in courses following these, while allowing the focus of the course to be on the conceptual understanding intrinsic to the reformed courses. However, logistical difficulties plague the pencil-and-paper administration of these tests, significantly decreasing their usefulness. To address these difficulties we have implemented an on-line version of the gateway tests which allows students to practice the skills on their own and take the test for a grade in a proctored environment. We report on the details of our implementation and assessment of our system, including our administration procedures and lab use. Our assessment included evaluation of students' perceptions of the effectiveness of the program and their actual skill acquisition, and showed that the students' skills improved in the course of the on-line gateway testing.

Key Words: Gateway testing, on-line testing, skills testing, assessment

2 Introduction

In this paper we report on the implementation issues for and results of the assessment of a web-based testing program designed to promote students' acquisition of algebraic and computational skills in precalculus and calculus courses at the University of Michigan. We first describe the context in which this program was implemented, including the historical background of reform at the University. Then we describe the program itself, including information on staffing and required computer resources. We next show the results of our assessment to determine its usability and effectiveness. Finally, we note some extensions to the system that we have put into place. This project was supported by the NSF CCLI Adaptation and Implementation grant program, grant #DUE-0088264, for which the second author (Megginson) was the Principal Investigator, and the implementer of the original paper-based gateway program at the University of Michigan described below. The first author (LaRose) conducted the development and implementation of the on-line gateway

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system.

3 Calculus Reform and Gateway Testing

The University of Michigan has since 1994 taught precalculus and calculus using a reformed textbook, technology, and cooperative group work [1]. In these courses there is a strong emphasis on mathematical concepts rather than on the skill-acquisition that is often prominent in more traditional courses. For example, our midterm and final exams in calculus include no questions of the standard type "find the [anti]derivative of the function $f(x) = \ldots$" However, even while we believe that our primary focus in these courses should not be on skill acquisition, we recognize that mathematicians, subsequent mathematics courses and courses in other disciplines all expect that students passing these courses will emerge with a well-defined skill set. Therefore, to ensure that students acquire these skills, we include gateway tests in the three reformed courses—precalculus, calculus I and calculus II. We currently have entrance gateway tests covering prerequisite skills for each of precalculus, calculus I and calculus II, and differentiation and integration gateways for calculus I and calculus II respectively. An example of each of these appears in the Appendix.

Gateway tests are skill-based tests on which students must be able to complete a set of 7–12 problems with very few (1–2) errors. Students are allowed to retake the tests throughout a several week testing period until this is accomplished. They are, however, limited to one attempt each day, which encourages them to stop and take the time to figure out what they missed before retaking the test. Because the tests are to prove technical facility with the skills being tested, they have a time cap of 20 minutes. Failure to pass a test by the deadline results in a reduction in the student's final letter grade for the course.[2] This reduction is between one-third and a full grade point (per gateway), as determined by the course-coordinator. We have found, however, that almost all students who do not pass the gateway are already doing failing work in the remainder of their coursework (individual and team homework, midterms and final).

The high standard required for a passing grade and allowance for multiple attempts mean that the gateway tests are more of a formative than summative assessment tool. Furthermore, because the gateway tests evaluate a specific skill set to an exacting standard we believe they are more effective than the alternative of including similar questions on the midterms or final, where missing one question would be indistinguishable from other errors on the exam.

As originally implemented, gateway tests were administered as pencil-and-paper tests in the department's Mathematics Tutoring Center. While this was preferable to requiring course instructors to manage them, it proved logistically difficult to sustain. At the University of Michigan we enroll over 1500 students in

[2] While this appears to be a strictly punitive grading scheme, it does serve to emphasize that the acquisition of these skills is regarded as an important but secondary side-effect of the course, rather than the primary component of the course goals.

calculus I in the Fall semester alone, with the result that as the deadline for the gateway test approached the demands on the Tutoring Center frequently exceeded its capacity: the number of tests that needed to be given, graded, processed and returned precluded this happening in a timely manner. As a result there were lines to get into the Tutoring Center and the length of time required for the tutors in the Center to be able to grade and return the tests increased from a day or two to several days or a week.

4 The On-Line Gateway Test

To address these logistical difficulties we are now offering all of our gateway tests on-line. This replaces the paper test with an on-line form into which students enter their answers after working out the problem by hand. When they have completed the test they submit it and the testing software automatically and immediately grades it and returns the result to the student. Because the test is on-line it is also possible for students to practice a test before coming to the department to take it for a grade.

When a student registers for one of the courses using the gateway tests, login data are created for him or her on the gateway server. In the week or so preceding the official gateway testing period the instructors in the course distribute this data, which include a login name and password, to the students in their classes. The students are then able to practice the gateway test from their dorm rooms or other University computer labs before being able to take the test for a grade. Once the testing period has begun, students come to a proctored lab in the mathematics department where their identities are verified and they then take the test in that controlled environment. Only those tests taken in the proctored lab count towards passing the gateway.

The actual gateway tests that are given on-line are drawn from testbanks that are similar to, but larger than, those which were used for the original pencil-and-paper test. Each testbank consists of a number of topics containing a large number of questions. A test is generated from the topic list by randomly selecting a predetermined number of questions from each topic. These are presented to the student in any order. Thus each student gets a different test every time he or she logs in to the system.

Each instructor is also given a login name and password for the system, which allows access to data showing which of the students in his or her class(es) have taken the test, and of those, which have passed. Because this interface to the system is also on-line, instructors are able to get instantaneously updated pass/fail data at any time and from any location.

5 Implementation Timeline

The on-line gateway testing system was implemented over the course of an academic year and exclusively employed starting in the following year. In Fall 2000 we tested two possible gateway testing systems in seven sections of calculus I, administering the differentiation gateway test in those classes. This involved 191 students and served to determine which of the software packages was best suited to our needs and what physical resources (number of computers, available lab space) we would need for full implementation of the system. The testing systems used were *WeBWorK* [2, 3], a free (for academic users) homework delivery system developed at the University of Rochester, and *eGrade* (now called *EDU*) [5, 6], a commercial testing system developed at the University of Nebraska and now marketed through Brownstone, Inc., and the John Wiley publishing company. Both systems allow students to enter their answers as regular mathematical expressions, and both correctly grade algebraically equivalent answers such as $4x+3$ and $4(x-1)+7$. (This is most important where multiple choice options can give away too much information, e.g., for antidifferentiation problems where multiple choice options are easily tested for correctness by differentiating them.) Because of its initial design as a testing system, we found it easier to adapt *eGrade* to our requirements for gateway testing, and we therefore used it in subsequent semesters.

As a result of this limited pilot we made three changes to the system for Winter 2001 (which is our regular "spring" term ending in April or May). A set of example problems demonstrating common syntactical errors students make when entering their answers was added for students to work through before taking the gateway test. Students were also allowed to retake the gateway test once a day if they did not pass on their first attempt, but only after they had gone over their test with a tutor in the Tutoring Center. And the length of time students were allowed to take the test was increased to 30 minutes, allowing them plenty of time to both work out their answers and correctly enter them in the computer interface.

In Winter 2001 we ran a pilot of the on-line gateway system with 11 sections each of calculus I (taking the differentiation gateway; the entrance gateway was not used that semester) and calculus II (taking the integration gateway; the entrance gateway was given on paper). These included 225 students in calculus I and 259 in calculus II. Assessment activities in this term included student surveys in calculus I classes using both the on-line and pencil-and-paper gateways, surveys in the calculus II courses using the on-line system, and surveys of instructors to determine how they perceived the on-line system. The results of this assessment appear below.

All of the gateway testing in Fall 2000 and Winter 2001 was done in the existing instructional computer labs in the department. The mathematics department has five computer labs, each with 15 entry-level Sun workstations. In Fall 2000 one of these labs was open six hours a day on weekdays (noon–4pm and 7–9pm) and two hours on Friday (noon–2pm) and Sunday (2–4pm). Students were given about two weeks to complete the gateway. For the Winter 2001 pilot we used up to two computer labs for gateway testing. This

proved sufficient to allow all students to take the tests with no or minimal (in the last couple of days before a deadline) waiting. The pilot ran without significant problems, allowing for the migration of all testing to the on-line gateway starting Fall 2001.

In Summer 2001 we installed a new computer lab of 30 computers to support the gateway testing and added a new instructional technology webserver to run the gateway testing software. The server was a four processor (350MHz Ultrasparc II) Sun with 1GB of memory which had previously served as a departmental computational server. This webserver was never taxed by the gateway testing system, suggesting that for the gateway testing alone a smaller server would suffice. The hours of the gateway lab were extended to 10am–4pm and 6–10pm Monday-Thursday, 10am–2pm Friday and 2–4pm Sunday, and two of the existing instructional computer labs were reserved as overflow labs for peak testing periods.

In Fall 2001 we had 120 classes with a total of about 3500 students using the gateway system. Approximately 8000 tests were given using the on-line system. In addition, two sections of calculus I at the University of Michigan Dearborn and three at Taylor University (Indiana) used the gateway tests from our server. In Winter 2002 approximately 70 classes with about 1500 students took their gateways on-line. In the Fall we found that in peak periods one or both of the overflow labs were required to avoid having students wait in significant lines. Because gateway tests are scheduled to avoid simultaneous deadlines, this translates to a peak need for between one and two computers available one hour a week for every student taking a gateway. This figure is for a gateway testing period of about two weeks, and corresponds to a computer to student ratio of approximately 1:25.[3]

6 Practical Issues

In this section we give some practical details on the implementation of the on-line gateway testing system.

6.1 Lab Organization and Proctors

We found that staffing the gateway testing labs required one proctor for every fifteen computers in the lab. Students coming in to take the gateway test in the proctored lab signed in on a log sheet at the door, were provided with blank paper on which to work, and logged in to the system at one of the lab computers. For the proctored test the gateway testing software requires a proctor login and password before the student can begin a gateway test, and the proctor checked the student's identity against a photo ID before authorizing the test.

On completing the gateway test the on-line system automatically graded the student's work and returned

[3]The University of Nebraska–Lincoln, using the same software, uses a ratio of approximately 1:30 in a lab that runs at very high capacity during peak periods. [7]

his or her result. If the student didn't pass he or she was urged by the proctors to print the graded test and go over it with a tutor in the Tutoring Center. After working with the student the tutor would sign and date (using a date-stamping machine) the printed copy of the test, thus providing evidence to the proctors in the gateway testing lab that the student was authorized to take the test a second time that day.

It is worth noting that having the sign-in sheets as a secondary record of students who came to the gateway lab proved a useful addition to the login record on the server on several occasions, because it provided a physical record of their (at least purportedly) being in the gateway testing lab.

Finally, a course-specific proctor login was created for each course using the gateway system. This allowed instructors to proctor tests in their offices for students in their courses.

6.2 Instruction of Instructors and Data Distribution

Instructors were introduced to the on-line gateway system at a course instructors' meeting in the week before students were to be allowed to start practicing the gateway. At this meeting they were given an overview of the system and the essential information needed to use it: the Web address their students needed to be able to practice the test on-line, the Web address where they could log in to get results for the students in their classes, their password for that, and a list of their students' login names and passwords.

The gateway system we are using (*eGrade*) can be configured to let students create their own system login data, thereby eliminating the need to build system rosters manually. However, we chose to disallow this, thereby ensuring that any students taking the gateway test were registered for the section of the course they were attending. This required that the login data be automatically generated from the course rosters, which we accomplished by writing an administration script to manage course and student data creation en masse from available course lists. (This and other administration scripts for the gateway were written and run by the department's instructional technology consultant, who is the gateway system administrator.)

Because we required that students be registered for a course section before creating their data on the gateway system, we also had to address the problem of students who added the course late or changed sections of the course after the testing period for the first (entrance) gateway had begun. This was done by writing another support script to manage such registration changes. The script verified that the student was enrolled in the correct course section by checking against current course rosters and then either created or moved the data record for the student appropriately. Instructors with students who added or changed into their class notified the gateway system administrator, who ran the script up to once a day to make the updates.

7 Assessment

Our assessment of the on-line gateway system sought to determine if it is effective at building the student skills, to assess student perception of the system (especially as compared to the pencil-and-paper version) and whether they felt they were learning as a result of having to take the gateway, and to assess instructor satisfaction with the system.

7.1 Students

At the end of the day, the effectiveness of the gateway system is determined by whether students do in fact learn the skills that we intend them to acquire by requiring them to pass the gateway tests, and whether their perception of the test is positive enough that they will use the system to acquire these skills.

When the traditional pencil-and-paper gateway tests were implemented at the University of Michigan in 1993, assessment of the effectiveness of the gateway indicated a strong correlation between students' having passed the gateway and having acquired the skills being tested on the gateway test [4]. To more directly measure this, in Winter 2002 we selected four sections of calculus I and tested students' acquisition of differentiation skills over the course of the gateway test through a pre- and post-test. The pre-test was given after the instructors had completed all instruction on the rules for finding derivatives, but before students had begun taking the differentiation gateway either for practice or for a grade. The post-test was given in the week or two following the end of the gateway testing period. Both pre- and post-tests were drawn from the gateway testbank, and consisted of four randomly selected functions which students had to differentiate. The pre- and post-tests were given as pencil-and-paper tests, and had a short time limit of 10 minutes.

In these four sections 94 students took the pre-test, 79 took the post-test, and 62 could be readily determined to have taken both (some of the pre- and post-tests were returned without names or ID numbers, preventing direct comparison of pre- and post-test results). In the table summarizing the test results, we refer to the last group as "pre- and post-test students." Tests were graded twice, once "rigorously," and once "leniently." For the "rigorous" grading, any answer that was not exactly correct was marked wrong—that is, the tests were marked in a manner similar to the computer-graded gateway tests. For the "lenient" grading errors attributable to a lack of precision in mathematical writing (such as dropped or parentheses in problems for which the calculation of the derivative was otherwise correct) were overlooked. These results are shown in Table 1.

Table 1 shows that there was a significant increase in students' average scores from the pre- to the post-test, and that the percent increase did not depend on the grading scheme employed. That is, the results suggest that there was an increase in students' skills at finding derivatives, but not their notational accuracy. This is a little surprising, as we expected that students would also improve the precision with which they

	rigorous grading		lenient grading	
	all students	pre- & post-test students only	all students	pre- & post-test students only
pre-test	1.31	1.40	1.48	1.56
post-test	2.65	2.66	2.95	2.97
%change	102%	90%	99%	90%

Table 1: Average scores on differentiation pre- and post-tests (out of four)

	rigorous grading		lenient grading	
	scores of 0 or 1	scores of 3 or 4	scores of 0 or 1	scores of 3 or 4
pre-test	56%	22%	51%	27%
post-test	13%	58%	9%	73%

Table 2: Distribution of scores on pre- and post-tests

write their answers—the on-line gateway test marks all answers with syntactical errors as incorrect, which should provide an incentive for students to improve in that area as well. The lack of notational improvement may be because the pre- and post-tests were done by hand while the gateway test was on-line and therefore required that students type their answers, but if this is the case it is then interesting that any improvement at typing answers did not translate into greater notational accuracy when working on paper.

We can also obtain a sense of the degree to which students' skills improved by looking at the distribution of scores. This is shown in Table 2, in which we show the percent of the students taking the pre- and post-tests whose scores demonstrated essentially no knowledge of the tested skills (scores of 0 or 1) and those who demonstrate significant knowledge (scores of 3 or 4). These results corroborate the data in Table 1, showing a significant improvement in students' ability with the tested skills.

Finally, we can consider the difference in scores of those students who could be readily ascertained to have taken both the pre- and post-tests. We found that 48 of the 62 (77%) scored higher on the post-test, while 8 (13%) scored lower. These figures are again consistent with the data presented in Tables 1 and 2.

From these results, it is clear that students' skills improved between the pre- and post-tests. The single biggest component of the course between the pre- and post-tests was the gateway test, so that we are fairly confident that this improvement may be attributed to students' efforts in response to the requirement that they pass the gateway test. The nature of our assessment is, however, unable to distinguish the effect of the gateway test and studying for it from that of other course activities and the work they demand of students. In particular, students were at the same time as they were taking the gateway test also working on homework assignments which required that they find derivatives. Furthermore, as students worked those assignments

it is not unlikely that their questions resulted in instructors briefly discussing basic differentiation. However, because of the relative weights assigned to these different assignments and because of the natures of the gateway test and the other assignments we still feel that it is reasonable to conclude that studying for the gateway test is the primary cause of students' improvement.

While we have only undertaken this formal assessment for one of the five gateway tests we have on-line, we expect similar results for the others.

Secondary assessment questions are concerned with the degree to which students' skills are evaluated well by the gateway test, and whether students feel that the requirement that they pass the gateway test results in them acquiring the skills. To assess these, we surveyed students in calculus I and II during the gateway pilot run in Winter 2001 and during the first full semester all gateways were on-line, Fall 2001. Surveys were distributed in class after the gateways were finished, and focused on two questions: (1) Did the students feel that they learned the skills on the gateway test as a result of having to pass it?, and (2) Did they feel that the skills that they had were evaluated well by the gateway test?

To be able to compare the on-line gateway and the pencil-and-paper version, in Winter 2001 we surveyed all calculus I courses that were using the on-line gateway and some others that were using the pencil-and-paper version. In addition, because that semester the calculus II courses involved in the pilot gave the entrance gateway on paper and the integration gateway on-line, we were able to ask students in those courses to compare the two testing methods. Finally, in an attempt to control for confounding variables, we also asked calculus I students if they were taking the course for the first time or had taken it before (e.g., in high school—in general we expect that students who are taking the course for a second time will think that they already know how to differentiate and will therefore be more likely to dispute that the gateway test is effective), and in Fall 2001 asked calculus II students if they had calculus I at Michigan (we expect that those having had calculus I will be more accustomed to the reformed calculus and gateway tests, and therefore give more positive responses).

The surveys asked students to rate from one to five their agreement (with a one indicating strong disagreement and a five strong agreement) with the statements "I learned the skills to take derivatives as a result of having to pass the gateway test," and "The gateway is a good way of evaluating student skills at finding derivatives." For calculus II, the word "derivative" was replaced with "integral."[4] The results of the surveys are shown in Table 3, which shows the average responses, broken down by question ("learned skills" and "good evaluation") and, within questions, by whether students had taken calculus I before or had calculus I at the University of Michigan ("%cal I" and "cal I" give results for student who, for calculus I

[4]That is, we focused on the integration gateway, which covers new material, rather than the entrance gateway. In Winter 2001 the assessment statements for the calculus II survey were slightly different, being instead "To what extent was the gateway a learning process that helped you learn integration skills" and "To what extent did your gateway score reflect your skill," with rating being from one (very little) to five (greatly).

				learned skills			good evaluation		
course	gateway	#students	%cal I	avg	cal I	no cal I	avg	cal I	no cal I
cal I–W01	on-line	138	45%	3.66	3.35	3.91	3.62	3.57	3.67
	paper	152	49%	3.88	3.84	3.92	3.55	3.63	3.48
cal II–W01	on-line	132	n/a	2.77			3.02		
	paper	132	n/a	2.86			3.53		
cal I–F01	on-line	80	71%	3.75	3.73	3.81	3.33	3.46	2.95
cal II–F01	on-line	397	24%	2.88	3.33	2.74	3.45	3.51	3.43

Table 3: Results of student surveys

classes, had calculus before or, for calculus II classes, who had calculus I at Michigan).

It is clear from Table 3 that students' perception of the degree to which they learned as a result of having to pass the gateway test fell somewhat short of that demonstrated in Tables 1 and 2. However, those students who had not taken calculus I prior to the semester in which the gateway survey was done indicated that the gateway test was effective at requiring them to learn the skills (as shown by the general agreement—scores of 3.9 and 3.8—with that statement). Calculus II students appear generally less convinced, so that it might be useful to do an assessment for the calculus II gateways similar to that run in calculus I to determine more objectively the effect of the gateway tests.

The student survey results also demonstrate that students' perception of the on-line test's effectiveness was equivalent to that of the pencil-and-paper version: this is most plainly seen by comparing the results for those students in calculus I in Winter 2001 who had not had calculus before. These results are essentially identical (3.91 and 3.92), and the results for Fall 2001 are similar (3.81). The responses of calculus II students are less clear, however. Students comparing the entrance gateway given on paper and integration gateway given on-line (in the same semester) somewhat preferred the paper gateway (with an average rating of 2.86 vs. 2.77), but comparing the results for students who used only the on-line gateway (from Fall 2001) shows that their assessment is equivalent to the previous semester's assessment of the paper gateway (2.88 vs. 2.86). It is a bit disturbing that the calculus II students' assessment of the gateway was generally negative (3.0 being neutral), and this may merit further study.

We note that on average students were more positive than not about the degree to which the gateway test was a good evaluation tool. This is fairly uniform regardless of the course being considered or the manner in which the gateway test was delivered—for calculus I it appears as if the the on-line test may be somewhat more highly regarded, a trend which is reversed for calculus II. The one negative score (2.95) here appears anomalous, but we do not have any explanation for its difference from the other results. As a whole, these scores reassure us that the gateway test results seen in Tables 1 and 2 do reflect actual student learning, even if the students are less convinced of this.

question	avg response	% positive responses
grading	4.44	88%
ease	4.19	81%
satisfied	4.13	81%

Table 4: Instructor survey responses

It is interesting that the survey results appear to bear out our expectation that students who are taking calculus I after having already had the course once before are less positive about the efficacy of the gateway tests, and that those students who go from our calculus I course to our calculus II course are generally more positive.

7.2 Instructors

To determine instructor satisfaction with the on-line system, we surveyed instructors teaching courses using the on-line test during our pilot in Winter 2001. Because all of the instructors had experience with the pencil-and-paper gateway test we did not also survey instructors who were not using the on-line system. From the instructors' perspective, we were interested in assessing whether they felt that the on-line system decreased their work-load, was easier to use, and was overall as effective as the pencil-and-paper gateway test. We also asked for their perspective on students' reaction to the on-line system.

The results of the survey are shown in Table 4. The questions asked were "Was your grading workload decreased by using the on-line gateway?" ("grading" in the figure), "Was the on-line gateway system easier to work with than the system of giving exams in the Math Lab?" ("ease" in the figure), and "Were you as satisfied with the way the on-line gateway worked as you were with the Tutoring Center?" ("satisfied"). To summarize responses answers were converted to a five point scale with 3 being neutral and 5 strong agreement. We make no distinction between instructors of calculus I and calculus II, as results for both were similar. The results include all 16 instructors teaching the sections using the on-line gateway tests.

None of the instructors' responses were negative. From these results it is clear that the instructors greatly preferred the on-line system, which should be expected given that the pencil-and-paper version frequently required instructors to grade tests when the Tutoring Center was swamped, and had a demonstrably long return time for tests taken at the end of the testing period.

We also asked the instructors "From your students' perspective, how well do you think the on-line gateway worked as compared to taking the gateway test in the Tutoring Center." Seven of the instructors indicated that they felt the two were equivalent, seven reported that their students had trouble with typing in their answers, two said that they thought it worked better than the pencil-and-paper gateway, and two referred

the question to the student survey results.

8 Conclusions

In the preceding we have described the on-line gateway testing system implemented at the University of Michigan. We have found that the on-line system addresses the logistical difficulties which were problematic with the pencil-and-paper testing we used before: all gateway tests are graded automatically by the on-line system, providing immediate feedback to the students taking the tests, and the capacity of the gateway testing computer lab(s) is sufficient to eliminate any significant waiting to take the tests even in peak times. As a result instructor work-load has been significantly decreased, as instructors no longer have to grade exams as the Tutoring Center becomes overloaded, and no longer have to hunt through stacks of gateway tests to determine which of their students have passed the test. Instructors also commented specifically that they highly valued the ability to use the on-line instructor interface to the system to determine at any time which of their students had passed and were attempting the gateway.

We have also assessed the testing system from the students' perspective. Student opinion surveys indicated that the on-line system is perceived as being at least as effective as the pencil-and-paper gateway test, and that students regard it more positively than not. We directly tested students' skills before and after they took a gateway test, and determined that the scores after the gateway increased significantly from those before. While we are unable to isolate the effect of studying for the gateway test from that of other course activities, we are reasonably confident that the increase may be primarily attributed to the requirement that students pass the gateway. We therefore conclude that our system as implemented "works."

We have also developed a tutorial system which automatically generates explanations for problems students miss when taking the gateway tests on-line. This allows students who are practicing the test to not only see immediately what problems they have answered incorrectly but also learn how to solve them correctly. We are in the process of assessing the effectiveness of this system.

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A Appendix: Sample Gateway Tests

The on-line gateway testing system described in this paper can be previewed on the Michigan Mathematics Department's Instructional Technology website, `<http://instruct.math.lsa.umich.edu/>`. Information about and content from the gateway testbanks developed through the NSF grant funding this work may also be obtained from this site. Note that calculator use is prohibited on all of the gateways.

A.1 Precalculus Entrance Gateway

1. The slope of the line having equation $-x - y - 2 = 0$ is

   (a) 2 (b) -2 (c) 0 (d) 1 (e) -1

2. The point $(-3, r)$ is on the graph of $y = x^3 - x^2v - 3$. What is v?

   (a) r (b) It cannot be determined from the information given (c) $\frac{-r-30}{9}$

   (d) $\frac{r+30}{9}$ (e) $\frac{-r-24}{9}$

3. Which of the following is equivalent to $k^4 + 2k^2 - 8$?

   (a) $(k+4)(k^3-2)$ (b) $(k+4)^2(k-2)^2$ (c) $(k^2+4)(k-\sqrt{2})(k+\sqrt{2})$

   (d) $(k+2)(k-2)(k^2+2)$ (e) $(k+2)^2(k-\sqrt{2})^2$

4. If $f(x) = a2^x$ then what is $\frac{f(-1.5)}{f(-0.5)}$?

   (a) 0.5 (b) -0.75×2^x (c) $2a$ (d) 0.75×2^x (e) $0.25a$

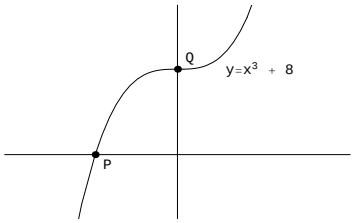
5. What is the greater of the two solutions to $3n^2 + 12n + 9 = 24$?

   (a) 1 (b) 3 (c) -1 (d) 5 (e) There are no real solutions to this equation

6. Which of the following is equivalent to $\frac{b^2+1}{\frac{1}{b}+1}$?

(a) b^3+1 (b) b^3 (c) $\frac{b^2+1}{b+\frac{1}{b}}$ (d) $\frac{b^3+b}{1+b}$ (e) $b+1$

7. What are the coordinates of the point P?



(a) $(2,0)$ (b) $(-2,0)$ (c) $(-8/3,0)$ (d) $(0,-2)$ (e) $(-1,0)$

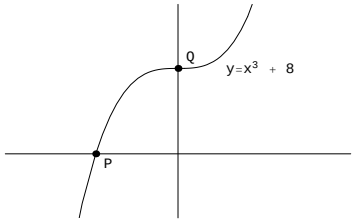
8. Which of the following is equivalent to $(A)(A^{2/9})$?

(a) A^2 (b) A^9 (c) $A^{11/9}$ (d) $A^{9/2}$ (e) None of these expressions are equivalent

9. Which of the following is equivalent to $\frac{a^7}{b^7}$?

(a) $(\frac{a}{b})^7$ (b) $\frac{a}{b}$ (c) $(ab)^0$ (d) $(ab)^{14}$ (e) None of these expressions are equivalent

10. If $P = (x, 12)$, what is the value of x?



(a) -0.2 (b) 15.2 (c) 12.8 (d) -20 (e) -140

11. If $y + x = (c - 4)c$ and the point $(-1, -3)$ is a solution, then what are all the possible values for c?

(a) 2 (b) -4 and 0 (c) 4 (d) 0 (e) 1 and -1

12. Suppose that $2x - y = 5$ and $x + 3y = -1$. What is x?

(a) 2 (b) -1 (c) -2 (d) 1 (e) 0

A.2 Calculus I Entrance Gateway

1. Find the domain of the function $f(x) = \frac{(x-9)(2x+1)}{4x+5}$.

(a) All real numbers (b) All real numbers except 9 and -1 (c) All real numbers except $-\frac{5}{4}$

(d) All real numbers except 9 and $-\frac{1}{2}$ (e) All real numbers between -1 and 9

2. If $g(x) = 4^x - x^4$, find $g(3)$.

(a) -17 (b) -5 (c) 5 (d) 47 (e) 49

3. If $F(x) = \sin x + \cos x$, find $F(b) - F(a)$ when $a = 0$ and $b = \pi$.

   (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

4. A regular hexagon with sides of length s has area $A = s^2(3\sqrt{3}/2)$. Express the perimeter p of the as a function of its area A.

   (a) $p = 6s$ (b) $A = \frac{\sqrt{3}p^2}{24}$ (c) $p = \sqrt{\frac{A}{54\sqrt{3}}}$ (d) $A = 54\sqrt{3}p^2$ (e) $p = \sqrt{8A\sqrt{3}}$

5. Find the slope of the line having y-intercept $-1/2$ and passing through the point $(7, -4)$.

   (a) $-13/8$ (b) $-1/2$ (c) $1/2$ (d) $13/8$ (e) $15/8$

6. Which of the following is equivalent to $\frac{z^{3n+3}}{3z^{-1}}$?

   (a) z^{n+1} (b) $3z^{3n+2}$ (c) $\frac{1}{3}z^{3n+2}$ (d) $\frac{1}{3}z^{3n+4}$ (e) $3z^{2n+2}$

7. Let $f(x) = 3x^2 - 7x$. Simplify $\frac{f(1+h)-f(1)}{h}$.

   (a) $3h - 2$ (b) $3h - 1$ (c) $3h$ (d) $3h + 1$ (e) $3h + 2$

8. Solve for y in the equation: $2y + 5 = x(3y - 4x)$

   (a) $\frac{2x^2}{3x^2-4x-5}$ (b) $\frac{5x^2}{-4x^2+3x-2}$ (c) $\frac{3x^2-2}{4x+5}$ (d) $\frac{4x^2+5}{3x-2}$ (e) $\frac{5x^2}{-4x^2-3x+2}$

9. Simplify the expression: $4z^2(2z^2)^2$

   (a) $32z^8$ (b) $16z^6$ (c) $32z^{12}$ (d) $8z^8$ (e) $24z^{10}$

10. Write the following as a single fraction: $x^{-5} - x^{-10}$

    (a) $\frac{x^5-1}{x^{10}}$ (b) $\frac{x^{10}-1}{x^5}$ (c) $\frac{1-x^5}{x^{10}}$ (d) $\frac{x^5+1}{x^{10}}$ (e) $\frac{x^{10}+1}{x^5}$

A.3 Calculus I Differentiation Gateway

1. Differentiate $f(x) = \sqrt{\ln x} + ex^2$
2. Differentiate $B(q) = \sqrt{\frac{2q^3-q-\pi}{2}} - q$
3. Differentiate $f(t) = e^{5t+2}(\frac{1}{2}t + \ln(2))$
4. Differentiate $f(t) = 2\pi + e^{(\pi t)}$
5. Differentiate $A(q) = \dfrac{\sin(4q) + \pi}{(q - \cos(4))^2}$
6. Differentiate $y = ax^r$, where a and r are constants
7. Differentiate $C(q) = \ln(e^2 + 5q)$

A.4 Calculus II Entrance Gateway

1. Evaluate the following : $\int_1^{e^2} \frac{1}{t}\, dt$.
2. Find the derivative of $f(x) = \frac{1}{2}(x^3 + 3x + \pi)^2$
3. Find the derivative of $y = e^t \sin(\pi t - 1)$
4. Find the derivative of $f(x) = \left(\dfrac{2}{\sin x}\right)^3$
5. If $y = (2 + x)\left(x^2 - 3\right)$, find the equation of the tangent line to the curve at $x = -1$.
6. Find the derivative of $g(p) = p \tan p$

A.5 Calculus II Integration Gateway

1. Find $\int \sin(3y - 4)\, dy$
2. Find $\int_0^{\ln(3)} e^y (e^y + 1)^2\, dy$
3. Find $\int_{-2}^{0} (8r^7 - r^3 + r)\, dr$
4. Find $\int \frac{\ln(7m-3)}{7m-3}\, dm$
5. Find $\int z^2 e^{3z}\, dz$
6. Find $\int 6\sqrt{1 - 3x}\, dx$
7. Find $\int_2^3 \frac{3x^6}{x^7-1}\, dx$